TUNABLE INTERSUBBAND ABSORPTION SPECTRUM BETWEEN LANDAU LEVELS IN GaAs/AlGaAs QUANTUM WELL

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Abstract: Intersubband optical absorption spectrum was studied in quantum well structures in quantizing magnetic field tilted with respect to the structure layers. Violation of the selection rule forbidding optical intersubband transitions in structures made of quantum wells with an asymmetric potential is proposed. The importance of asymmetric structure design to achieve considerable values of optical absorption is demonstrated.
Keywords: Intersubband absorption; quantum well; Landau levels; tilted magnetic.

1. Introduction

Recently, the possibility of achieving population inversion in the system of Landau levels (LL) in cascade structures of quantum wells in a strong magnetic field under the condition of sequential resonant tunneling, that is, in a strong transverse electric field, has been shown [1]. In the case the optical phonon scattering is suppressed, i.e. the separation between first and any upper (ν-th) subbands energy levels is less than the optical phonon energy, the population in the zero LL in the ν-th subband may exceed the population of the first LL in the first subband. Thus, stimulated radiation can be achieved at transitions between these two levels, and the radiation frequency can be continuously adjusted over a wide range of terahertz frequencies by changing the magnetic field strength in accordance with the expression

$$\hbar\omega = \Delta E_{1\nu} - \hbar\omega_C \quad (1)$$

where $\Delta E_{1\nu}$ is the energy spacing between the first and ν-th subband, ω_C is the cyclotron frequency. The scheme of transitions between the Landau levels of subbands 1 and 2 in the quantum well structure considered in [1] is shown in Fig. 1. The main problem is that in a magnetic field directed perpendicular to the layers of the structure, the optical transition of interest (2,0)→(1,1) (shown in Fig. 1) is forbidden [2, 3], that is, the corresponding dipole matrix element is exactly zero.

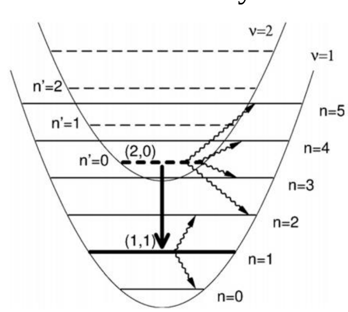

Figure 1: *A scheme of transitions between Landau levels in a quantum well. The thick arrow indicates the (2, 0)→(1,1) radiative transition, and the wavy arrows mark the transitions due to the electron-electron scattering [1].*

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In this paper, we investigated the effect of the tilted magnetic field slope on the optical matrix element of intersubband transitions. The importance of an asymmetric structure for achieving significant values of transition dipole matrix elements was revealed, and an asymmetric two-well structure was proposed as a possible solution, which allows maximizing the optical matrix element of the optical intersubband transitions of interest.

2. Theoretical background

Consider electron in the quantum well structure with potential profile $V_{QW}(z)$ in the tilted magnetic field $\vec{B} = B_{\|}\vec{e}_y + B_{\perp}\vec{e}_z$, where z is the growth axis. In Landau gauge $\vec{A} = (B_{\|}z - B_{\perp}y)\vec{e}_x$ envelope wave function of electron is given by [2]

$$\psi(x,y,z) = \frac{\exp(ikx)}{\sqrt{L}} f(y - k\ell_{\perp}^2, z). \tag{2}$$

Here function $f(y,z)$ is determined by two-dimensional Schrodinger equation with Hamiltonian $\hat{H}_{2D} = \hat{H}_{\perp} + \hat{H}_{tilt}$. Where

$$\hat{H}_{\perp} = -\frac{\hbar^2}{2m(z)}\frac{\partial^2}{\partial y^2} + V_{QW}(z) - \frac{\partial}{\partial z}\frac{\hbar^2}{2m(z)}\frac{\partial}{\partial z} + \frac{\hbar^2}{2m(z)}\frac{y^2}{\ell_{\perp}^4} \tag{3}$$

is the Hamiltonian in the case of magnetic field $\vec{B} = B_{\perp}\vec{e}_z$ normal to the structure layers, and

$$\hat{H}_{tilt} = \frac{\hbar^2}{2m(z)}\frac{z^2}{\ell_{\|}^4} - \frac{\hbar^2}{m(z)\ell_{\perp}^2\ell_{\|}^2} yz. \tag{4}$$

Here $m(z) = \begin{cases} m_w & (z \in well) \\ m_b & (z \in barrier) \end{cases}$ is the effective mass, $\ell_{\perp} = \sqrt{\frac{\hbar c}{eB_{\perp}}}$ and $\ell_{\|} = \sqrt{\frac{\hbar c}{eB_{\|}}}$ are the magnetic lengths for transverse ($B_{\perp}$) and longitudinal ($B_{\|}$) magnetic field components.

If magnetic field $\vec{B} = B_{\perp}\vec{e}_z$ being perpendicular to the structure layers, the variables in the Schroedinger equation are separated, energy levels and wave functions of Landau states are given by [3]

$$E_{(\nu,n)} = \varepsilon_{(\nu,n)} + \hbar\omega_{\perp}(n + \tfrac{1}{2}) \tag{5}$$

$$f_{\nu,n}(y,z) = \varphi_{\nu}(z)\Phi_n(y) \tag{6}$$

where $\Phi_n(y)$ is the wave function of harmonic oscillator with mass m_w and frequency $\omega_{\perp} = eB_{\perp}/(m_w c)$, $\varepsilon_{(\nu,n)}$ and $\varphi_{\nu}(z)$ are the energy and wave function of ν th subband.

It can be easily seen that in this case, the dipole matrix element

$$\vec{D}_{(1,0)\to(2,n)} = \left\langle \frac{\exp(ikx)}{\sqrt{L}} f_{(1,0)}(y - k\ell_{\perp}^2, z) \middle| \vec{r} \middle| \frac{\exp(ikx)}{\sqrt{L}} f_{(2,n)}(y - k\ell_{\perp}^2, z) \right\rangle \tag{7}$$

is exactly equal to zero for any polarization due to the orthogonality of oscillator wave functions $\left\langle \Phi_{n_1}(y) \middle| \Phi_{n_2}(y) \right\rangle = \delta_{n_1,n_2}$, that is, the considered $(1,0) \to (2,n)$ optical transition is forbidden.

However, the matrix element of the specified transition can be made nonzero by applying an additional component $B_\parallel$ of the magnetic field parallel to the layers, that is, by tilting the magnetic field with respect to the structure layers. Due to an additional term

$$\hat{H}_{mix} = -\frac{\hbar^2}{m(z)\ell_\perp^2 \ell_\parallel^2} yz \tag{8}$$

the variables in the Schrodinger equation are no longer separated, because of mixing in-plane and out-of-plane electron motions [4], and lifting of the above selection rule.

Consider the situation when the matrix element of the Hamiltonian (4) over the first and second subband stated (6) is much lower than the subband spacing. This is the case in the magnetic field range when $\hbar\omega_\perp < E_2 - E_1$. The structure of a single-electron spectrum in the tilted magnetic field in this case does not change significantly [4]. The main effect of $B_\parallel$ is the shift of the harmonic oscillator center in equation (6) by the value $\frac{\ell_\perp^2}{\ell_\parallel^2}\langle z\rangle_\nu$, where $\langle z\rangle_\nu = \int |\varphi_\nu(z)|^2 zdz$ is the average value of the electron coordinate along the z axis in the ν-th subband state [5]

$$f_{\nu,n}(y,z) = \varphi_\nu(z)\Phi_n\left(y - \tfrac{\ell_\perp^2}{\ell_\parallel^2}\langle z\rangle_\nu\right), \tag{9}$$

and Landau level (ν, n)

$$E_{(\nu,n)} = \varepsilon_\nu + \hbar\omega_\perp(n+1/2) + \frac{m_w\omega_\parallel^2}{2}\left(\langle z^2\rangle_\nu - \langle z\rangle_\nu^2\right) \tag{10}$$

Substituting wave functions (9) into equation (7), the following expression can be obtained for the squared modulus of the dipole matrix element:

$$\left|\vec{D}_{(1,0)\to(2,n)}\right|^2 = \delta_{k_1,k_2}\left|\langle\varphi_2(z)|z|\varphi_1(z)\rangle\right|^2 F_n(\xi). \tag{11}$$

Where

$$F_n(\xi) = \left(\frac{\xi^{2n}}{2^n n!}\right)\exp\left(-\frac{\xi^2}{2}\right), \tag{12}$$

$$\xi = \left[\langle z\rangle_2 - \langle z\rangle_1\right]\frac{\ell_\perp}{\ell_\parallel^2}. \tag{13}$$

We can see that the dipole matrix element becomes nonzero only if the values $\langle z\rangle_2$ and $\langle z\rangle_1$ are substantially different. In quantum well with symmetric potential, the subband wave functions $\varphi_\nu(z)$ are symmetric or antisymmetric with respect to symmetry center of the potential, and the averages $\langle z\rangle_\nu$ are the same for all subbands. So, in symmetric potential, the transition matrix element continues to be close to zero even in the tilted magnetic field. Thus, to provide a nonzero dipole matrix element for transitions of interest along with the application of the tilted magnetic field, it is necessary to introduce an asymmetric potential along the direction of the structure growth.

3. Intersubband absorption in quantum well structures with asymmetric potential

The optical transition is described in the dipole approximation [6] by using the Fermi rule. This leads to the following expression for the number of transition events per unit time from the Landau level $i=(\nu_i,n_i)$ to the Landau level $f=(\nu_f,n_f)$, induced by the absorption of the phonon $\hbar\omega$

$$W_{i\to f}(\omega)=I(\omega)L^2\frac{4\pi^2e^2}{c\hbar\eta}\frac{\left(E_f-E_i\right)^2}{\left(\hbar\omega\right)^2}\frac{2}{\alpha L^2}\sum_k\left|\vec{\varepsilon}\vec{D}\right|^2\left[N_i-N_f\right]\delta\left(E_f-E_i-\hbar\omega\right). \quad (14)$$

Here $I(\omega)$ is the intensity of the incident radiation, η is refractive index, $\boldsymbol{\varepsilon}$ is the vector of radiation polarization, N_i is the population of the Landau level *i*, $\alpha=1/(\pi\ell_\perp^2)$ is degeneracy of Landau level.

We characterize the $i=(1,0)\to f=(2,n)$ transition absorption by the coefficient $\alpha_{i\to f}(\omega)$, which is defined as the ratio of the power of radiation absorbed on this transition and power of incident radiation on the sample, i.e.

$$\alpha_{i\to f}(\omega)=\frac{\hbar\omega W_{i\to f}(\omega)}{I(\omega)L^2} \quad (15)$$

The peak of $\alpha_{i\to f}(\omega)$ occurs at the frequency

$$\begin{aligned}\hbar\omega_0=E_f-E_i&=\varepsilon_2-\varepsilon_1+n\hbar\omega_\perp+\frac{m_w\omega_\parallel^2}{2}\left(\left\langle z^2\right\rangle_2-\left\langle z^2\right\rangle_1-\left\langle z\right\rangle_2^2+\left\langle z\right\rangle_1^2\right)\\&=\Delta\varepsilon+\hbar\omega_\perp\left\{n+\left(\frac{\left[(\delta z)_2\right]^2}{(\Delta z)^2}-\frac{\left[(\delta z)_1\right]^2}{(\Delta z)^2}\right)\frac{\xi^2}{2}\right\}\end{aligned} \quad (16)$$

where $\Delta\varepsilon=\varepsilon_\nu-\varepsilon_1$ is intersubband spacing, $\delta z_\nu=\sqrt{\left\langle z^2\right\rangle_\nu-\left\langle z\right\rangle_\nu^2}$ is root mean square fluctuation of the electron coordinate and $\Delta z=\left\langle z\right\rangle_2-\left\langle z\right\rangle_1$.

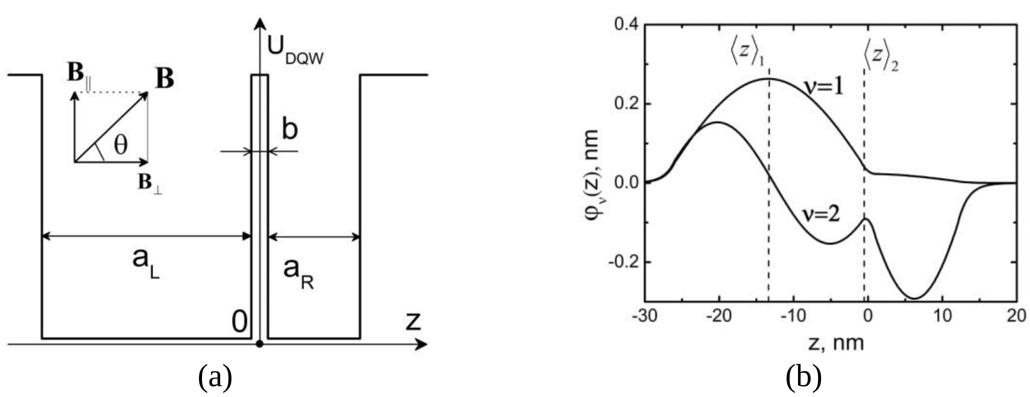

Figure 2: *The potential profile of asymmetrical double quantum well (a) and respective subband wave functions (b).*

The amplitude of peak is determined by both the factor of delta-function (in the following, the power of the absorption line)

$$a_{i\to f}(\omega)=\frac{4\pi^2 e^2}{c\hbar\eta}\hbar\omega_0\left|\langle\varphi_2(z)|z|\varphi_1(z)\rangle\right|^2 F(\xi)\left[N_i-N_f\right] \qquad (17)$$

and its width (as a rule, it is proportional to the half-width Γ). However, the effects discovered in the presented work are caused by features contained in the dipole matrix element $\vec{D}$, and are not principally affected by the peak width. Therefore, for illustrative purposes, to get the scale, we use the Gaussian shape of absorption line with a typical value of Γ=1 meV.

As it is seen from the obtained expressions, the contribution of the longitudinal component $B_{\|}$ of the magnetic field is determined by the parameter ξ which is proportional to the difference of the average coordinates of electron along the structure growth axis in initial and final states. This causes the substantially different effect of this component on absorption depending on the symmetry of the quantum well structure potential.

In the case of asymmetric structures from quantum wells, the parameter ξ may become non-zero when the potential profile is selected accordingly. Nonzero values of ξ can lead to a significant violation of the selection rule $\Delta n=0$ in a tilted magnetic field. However, the question of how this violation of the selection rules affects the intensities of all possible optical transitions between Landau levels in such structures is of considerable interest.

First and foremost, it is easy to see that the ratio of amplitudes transition with $\Delta n\neq 0$ and $\Delta n=0$ is

$$\frac{a_{(1,0)(2,n)}}{a_{(1,0)(\nu,0)}}=\left[1+\frac{\hbar\omega_\perp}{\Delta\varepsilon+\frac{m_w}{2\hbar^2}\left[(\delta z_2)^2-(\delta z_1)^2\right](\hbar\omega_\|)^2}n\right]\cdot\frac{\xi^{2n}}{2^n n!} \qquad (18)$$

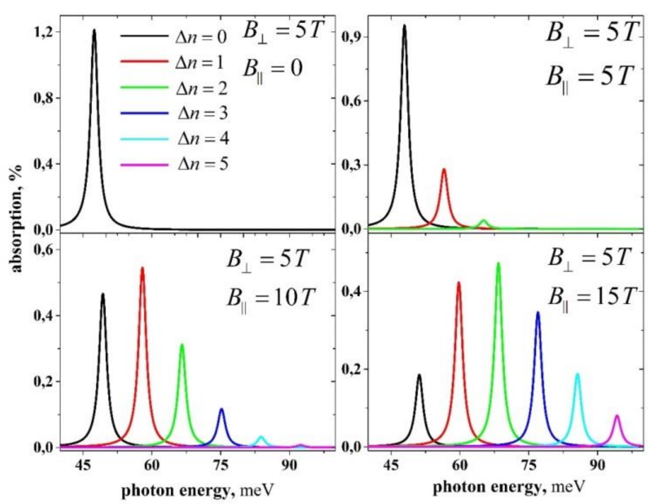

Figure 3: *The absorption spectrum in asymmetrical structure on (1,0) → (2,Δn) transitions with the fixed $B_\perp=5T$ and different values of $B_\|$*

It is clear that the expression in brackets is always greater than 1. The parameter $\xi \propto \frac{B_{||}}{\sqrt{B_{\perp}}}$, so it can be varied by changing the magnitude and orientation of the magnetic field. Accordingly, when $\xi > \sqrt[2n]{2^n n!}$ the right-hand side (18) will be obviously greater than 1, i.e. the intensity of optical transitions with $\Delta n \neq 0$ will dominate.

The value $\hbar\omega_{0,n}$ is a monotone function of ξ, while the function $F_n(\xi)$ has an explicit maximum at $\xi = \sqrt{2n}$. This leads to the following general behavior of line absorption intensity. For $\xi = 0$, there is only the line in the spectrum corresponding to the optical transition with $\Delta n = 0$. As soon as ξ becomes non-zero, the absorption lines with $\Delta n \neq 0$ appear and their intensities grow when ξ increases. When $\xi \approx \sqrt{2}$, the intensity of a line from $\Delta n = 1$ reaches the maximum. The intensity of this line decreases with the further increase in ξ. However, the line amplitude from $\Delta n = 2$ rises and reaches the maximum when $\xi \approx 2$, then goes down, and so on. Besides, the intensity of line corresponding to $\Delta n = 0$ decreases close to exponential.

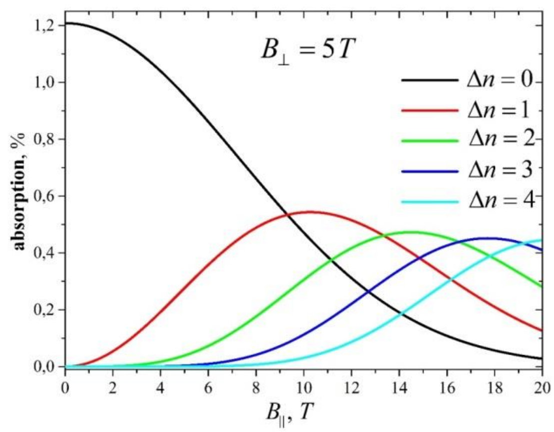

Figure 4: *The line intensity dependence on* $B_{||}$ *when* $B_{\perp}$ = *1T*

This is illustrated in Fig. 3, which shows the absorption spectra at the transitions between the Landau levels of the first and second subband calculated for an asymmetric structure of two tunnel-coupled GaAs/$Al_{0.3}Ga_{0.7}As$ quantum wells of different widths (12 and 5 nm) separated by a 2 nm barrier (Fig. 2). The calculation is performed for a fixed component $B_{\perp}$, and the parameter ξ is changed by a changing $B_{||}$. It is shown that when $B_{||} = 0$ (magnetic field $\mathbf{B} = B_{\perp}\mathbf{e}_z$ being perpendicular to the structure layers) in the spectrum there is only the line with $\Delta n = 0$, corresponding to the optical transition (1,0)→(2,0), because of selection rule. As soon as the value of $B_{||}$ becomes non-zero in the spectrum appears absorption lines with $\Delta n \neq 0$ due to the breaking of violation of the selection rule and we can shift the absorption lines at transitions with $\Delta n \neq 0$ towards higher frequencies with a maximum absorption intensity. The dependences of the absorption line intensities on $B_{||}$ are shown in Fig. 4. It is shown that the absorption intensity on the optical transitions with $\Delta n \neq 0$ can exceed the absorption intensity on the transition with $\Delta n = 0$. So, by changing the magnetic field, we can control the absorbed frequency with a maximum absorption intensity.

4. Conclusion

In this work we studied the intersubband absorption spectrum behaviour in quantum well structures in arbitrarily directed magnetic field. The violation of the $\Delta n = 0$ selection rule and the appearance of additional absorption lines corresponding to transitions with $\Delta n \neq 0$ was demonstrated in structures with asymmetric potential in tilted magnetic field. The intensities of these transitions can substantially exceed that of the $\Delta n \neq 0$ transition.

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TÓM TẮT

KHẢ NĂNG ĐIỀU KHIỂN PHỔ HẤP THỤ GIỮA CÁC MỨC LANDAU CỦA GIẾNG LƯỢNG TỬ GaAs/AlGaAs

Trong bài báo này, chúng tôi nghiên cứu phổ hấp thụ giữa các vùng con trong giếng lượng tử đặt trong từ trường có phương nghiêng so với trục các lớp bán dẫn. Sự vi phạm quy tắc lọc lựa để đạt được các dịch chuyển quang học mong muốn có thể đạt được trong các cấu trúc từ giếng lượng tử có thế năng bất đối xứng. Vai trò quan trọng của các cấu trúc bất đối xứng để nâng cao hệ số hấp thụ trên các dịch chuyển quang học quan tâm được chứng minh.

Từ khóa: Hấp thụ giữa các vùng con; giếng lượng tử; mức Landau; từ trường nghiêng.