THE NUMBER OF MAXIMAL INDEPENDENT SETS OF SEQUENTIALLY COHEN-MACAULAY BIPARTITE GRAPHS

Dao Thi Thanh Ha
Vinh University

Received on 05/5/2021, accepted for publication on 19/7/2021

Abstract: We determine the maximum number of maximal independent sets of sequentially bipartite graphs and we give a completed characterization of the extremal graphs.

Keywords: Graph; independent set; sequentially Cohen-Macaulay.

1 Introduction and results

Let $\mathrm{m}(G)$ be the number of maximal independent sets of a simple graph G. Around 1960, Erdös and Moser raised the problem of determining the largest number of $\mathrm{m}(G)$ in terms of its order, say n in this paper, and determining the extremal graphs. In 1965, Moon and Moser [11] solved this problem for any simple graph.

This problem now has been focused on various classes of graphs (see e.g. [3, 9. 10, 15]). For the simplest case, Wilf [15] was the first to to prove that if T is tree with n vertices, then

$$\mathrm{m}(T) \leqslant \begin{cases} 2^{s-1}+1 & \text{if } n = 2s, \\ 2^s & \text{if } n = 2s+1 \end{cases}$$

and he also characterize those trees achieving the maximum value.

The goal of this paper is to extend this result to sequentially Cohen-Macaulay bipartite graphs. Let G be a simple (no loops or multiple edges) undirected graph on the vertex set $V(G) = \{1, \dots, n\}$. Let K be a field and $R = K[x_1, \dots, x_n]$ the polynomial ring over K. Then, we can associate to G a quadratic square-free monomial ideal

$$I(G) = (x_i x_j \mid \{i,j\} \in E(G)) \subset R,$$

where $E(G)$ is the edge set of G. The ideal $I(G)$ is called the *edge ideal* of G. Using the Stanley-Reisner correspondence, we can associate to G the simplicial complex $\Delta(G)$ where $I_{\Delta(G)} = I(G)$.

Notice that the faces of $\Delta(G)$ are the independent sets or stable sets of G, i.e. S is a face of $\Delta(G)$ if and only if there is no edge of G joining any two vertices of S. Thus, $\mathrm{m}(G)$ is just the number of maximal sets of $\Delta(G)$ (with respect to inclusion).

Note that the property of being sequentially Cohen-Macaulay, a condition weaker than being Cohen-Macaulay, was introduced by Stanley [13] in connection with the theory of nonpure shellability. A graded R-module M is called sequentially Cohen-Macaulay (over k) if there exists a finite filtration of graded R-modules

$$0 = M_0 \subset M_1 \subset \cdots \subset M_r = M$$

1) Email:thahanh@yahoo.com

such that each M_i/M_{i-1} is Cohen-Macaulay, and the Krull dimensions of the quotients are increasing:

$$\dim(M_1/M_0) < \dim(M_2/M_1) < \cdots < \dim(M_r/M_{r-1}).$$

We say that a graph G *sequentially Cohen-Macaulay* if $R/I(G)$ is sequentially Cohen-Macaulay. The class of sequentially Cohen- Macaulay graphs have been interested by many authors (see [4, 5, 6, 14]).

The main result of this paper is to extend the result of Wilf to the sequentially Cohen-Macaulay bipartite graphs. Recall that a graph G is bipartite if the vertex set $V(G)$ can be partitioned into two disjoint sets $V(G) = V_1 \cup V_2$ such that every edge of G contains one vertex in V_1 and the other in V_2. The couple (V_1, V_2) is called a bipartition of G.

Example 1.1. *All trees are sequentially Cohen- Macaulay (see [14, Theorems 2.13 and 3.10].*

For integers $i \geqslant 1$ and $j \geqslant 0$, we define the *baton* $B(i, j)$ to be the graph obtained from a basic path P of i vertices by attaching j paths of length two to the endpoints of P (see Figure 1 below).

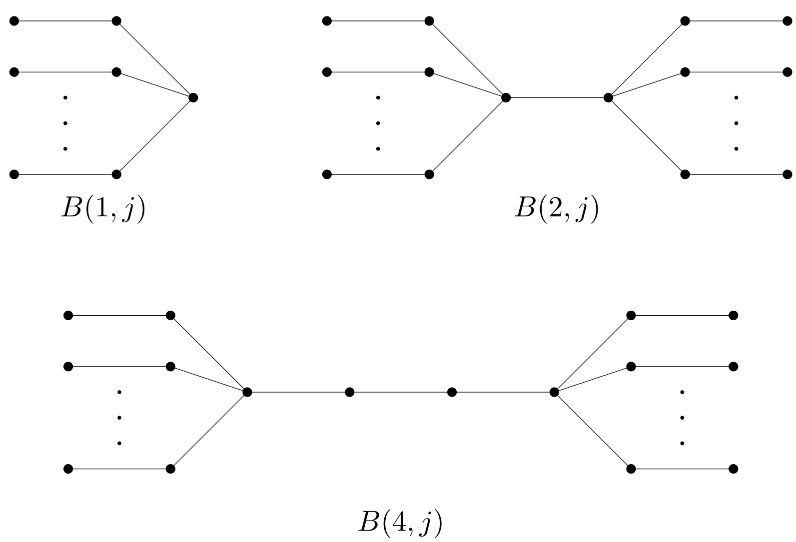

Fig. 1: *Batons.*

Then, the main result of the paper is the following theorem.

Theorem 3.2. *Let G a connected sequentially Cohen-Macaulay bipartite graph with n vertices. Then,*

$$\mathrm{m}(G) \leqslant f(n) = \begin{cases} 2^s & \text{if } n = 2s+1, \\ 2^{s-1}+1 & \text{if } n = 2s. \end{cases}$$

Furthermore, $\mathrm{m}(G) = f(n)$ *if and only if*

$$G \cong \begin{cases} B(1,s) & \text{if } n = 2s+1, \\ B(2,s-1) \text{ or } B(4,s-2) & \text{if } n = 2s. \end{cases}$$

2 Preliminaries

Let G be a simple graph with vertex set $V(G)$ and edge set $E(G)$. An edge $e \in E(G)$ connecting two vertices x and y will be also written as xy (or yx). In this case, it is said that x and y are adjacent. A set of vertices of G is *independent* if every pair of its vertices is not adjacent.

A path P of G is a sequence of vertices

$$P\colon v_0, v_1, \ldots, v_k$$

such that $v_{i-1}v_i$ is an edge of G for $i = 1, \ldots, k$. Then, we say that P connects two vertices u and v; and k is the length of P.

A cycle in the graph G is a non-empty path in which the only repeated vertices are the first and last vertices. The length of a cycle is the number of edges involved

A connected graph without cycles is called a tree.

A graph G is *connected* whenever there is a path between every pair of vertices. A graph is called *totally disconnected* if it is either a null graph or it contains no edge. If G is totally disconnected graph, then $\mathrm{m}(G) = 1$.

Let v be a vertex of G. The neighborhood of v in G is the set

$$N_G(v) = \{u \in V(G) \mid uv \in E(G)\}.$$

The number $\deg_G(x) = |N_G(x)|$ is called the *degree* of x in G. If $\deg_G(v) = 0$, then v is called an *isolated vertex* of G; and if $\deg_G(v) = 1$, then v is called a *leaf* of G.

For a subset S of $V(G)$, the subgraph of G obtained from G by removing all vertices in S and their incident edges, denoted by $G \setminus S$. The graph $G \setminus (V(G) \setminus S)$ is called the *induced subgraph* of G on the vertex S, and denoted by $G[S]$.

For a vertex v of G, we denote $G \setminus v = G \setminus \{v\}$ and $G_v = G \setminus (\{v\} \cup N_G(v))$.

Lemma 2.1. [15] *If* $\mathcal{H}$ *is an induced subgraph of* G*, then*

$$\mathrm{m}(\mathcal{H}) \leq \mathrm{m}(G).$$

Lemma 2.2. [9, Lemma 1] *Let* G *be a graph. Then*

1. $\mathrm{m}(G) \leq \mathrm{m}(G_v) + \mathrm{m}(G \setminus v)$*, for any vertex* v *of* G*.*
2. *If* v *is a leaf adjacent to* u*, then* $\mathrm{m}(G) = \mathrm{m}(G_v) + \mathrm{m}(G_u)$*.*

Lemma 2.3. [14, Lemma 2.8] *If* G *is a sequentially Cohen-Macaulay bipartite graph, then there is* $v \in V(G)$ *with* $\deg_G(v) = 1$*.*

A *matching* in G is a set M of edges so that no two of which meet a common vertex. The *matching number* $\nu(G)$ of G is the maximum size of matchings of G. If every vertex of G is incident to an edge of M, then M is called a perfect matching. Note that $|V(G)| \geqslant 2\nu(G)$ and the equality occurs if and only if G has a perfect matching.

An *induced matching* M in a graph G is a matching where no two edges of M are joined by an edge of G. The *induced matching number* $\nu_0(G)$ of G is the maximum size of induced matchings of G. We always have $\nu_0(G) \leq \nu(G)$; and if $\nu_0(G) = \nu(G)$ then G is called a *Cameron-Walker* graph after Hibi et al. [7].

Cameron and Walker [2] gave a classification of the simple graphs G with $\nu(G) = \nu_0(G)$; such graphs now are the so-called Cameron-Walker graphs (see [7]).

Lemma 2.4. ([2, Theorem 1] or [7, p. 258] *A graph G is Cameron-Walker if and only if it is one of the following graphs:*

1. *a star;*
2. *a star triangle;*
3. *a finite graph consisting of a connected bipartite graph with bipartition (A, B) such that there is at least one leaf edge attached to each vertex $i \in A$ and that there may be possibly some pendant triangles attached to each vertex $j \in B$.*

Example 2.5. *Let G be Cameron-Walker graph with 8 vertices in Figure 1. Then $\nu(G) = \nu_0(G)2$ and the maximal independent sets of G are*

$$\{1, 2, 5, 6, 7, 8\}; \{3, 4\}; \{3, 5, 6\}; \{4, 7, 8\}$$

Hence, $\mathrm{m}(G) = 4$.

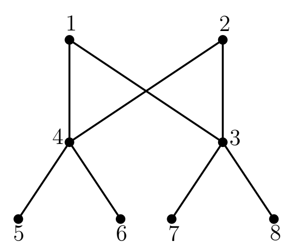

Fig. 2: *Cameron-Walker bipartite graph.*

Lemma 2.6. *Let G be a bipartite graph. Then $\mathrm{m}(G) \leq 2^{\nu(G)}$. Furthermore, the equality occurs if and only if G is a Cameron-Walker bipartite graph.*

Proof. Follows from [8, Corollary 3.4]. □

3 The proof of the main result

We begin with the following lemma.

Lemma 3.1. *Let $r, s_1, \dots, s_r$ be positive integers. Then*

$$\prod_{i=1}^{r}(2^{s_i-1}+1)+\prod_{i=1}^{r}2^{s_i-1}\le 2^{\sum_{i=1}^{r}s_i}+1$$

and the equality only when $s_1=\cdots=s_r=1$ or $r=1$.

Proof. We prove by the induction on r. If $r=1$, then it is obvious.

If $r>1$, then we assume that $s\le s_i$ for all $i=1,\cdots,r$. We will show that

$$(2^{s-1}+1)\prod_{i=1}^{r}(2^{s_i-1}+1)+2^{s-1}\prod_{i=1}^{r}2^{s_i-1}\le 2^{s+\sum_{i=1}^{r}s_i}+1.$$

We have

$$\begin{aligned}
&(2^{s-1}+1)\prod_{i=1}^{r}(2^{s_i-1}+1)+2^{s-1}\prod_{i=1}^{r}2^{s_i-1}\\
=\ &(2^{s-1}+1)\Big[\prod_{i=1}^{r}(2^{s_i-1}+1)+\prod_{i=1}^{r}2^{s_i-1}\Big]-\prod_{i=1}^{r}2^{s_i-1}\\
\le\ &(2^{s-1}+1)(2^{\sum_{i=1}^{r}s_i}+1)-\prod_{i=1}^{r}2^{s_i-1}\\
\le\ &\Big[2^{s+\sum_{i=1}^{r}s_i}+1\Big]+2^{\sum_{i=1}^{r}s_i}(1-2^{s-1})+(2^{s-1}-2^{\sum_{i=1}^{r}s_i-r}).
\end{aligned}$$

Since $1\le s\le s_i$ for all i and $(s-1)\le r(s-1)\le\sum_{i=1}^{r}s_i-r$, we have $1-2^{s-1}\le 0$ and $2^{s-1}\le 2^{\sum_{i=1}^{r}s_i-r}$. Thus, inequality is proved.

Now the equality holds only when $r=1$ or $s_1=\cdots=s_r=1$. This completes the proof of the lemma. □

The main result of this paper is to establish the maximum value of $\mathrm{m}(G)$ for a connected sequentially Cohen-Macaulay bipartite graph G.

Theorem 3.2. *Let G a connected sequentially Cohen-Macaulay bipartite graph with n vertices. Then,*

$$\mathrm{m}(G)\leqslant f(n)=\begin{cases}2^s & \text{if } n=2s+1,\\ 2^{s-1}+1 & \text{if } n=2s.\end{cases}$$

Furthermore, $\mathrm{m}(G)=f(n)$ *if and only if*

$$G\cong\begin{cases}B(1,s) & \text{if } n=2s+1,\\ B(2,s-1)\ \text{or}\ B(4,s-2) & \text{if } n=2s.\end{cases}$$

Proof. We first consider the case n is odd, i.e. $n=2s+1$ for some $s\geqslant 0$. Note that $\nu(G)\leqslant s$, so that $\mathrm{m}(G)\leqslant 2^s$ by Lemma 2.6. Furthermore, the inequality occurs if and only if $\nu(G) = \nu_0(G) = s$. It follows that G has an induced matching with s edges, say $M = \{a_1b_1, \ldots, a_sb_s\}$.

Since $|V(G)| = 2s+1$, there is another vertex v which is not incident to any edge in M. Note that G is connected, we may assume that v is adjacent to a_i for all i. Since G is bipartite, v is not adjacent to b_i for all i. This means that $G = B(1,s)$, and the theorem is proved for this case.

Assume that n is even, i.e. $n = 2s$ for some $s \geqslant 1$. Note that $\nu(G) \leqslant s$. If $\nu(G) < s$, then $\mathrm{m}(G) \leqslant 2^{\nu(G)} \leqslant 2^{s-1}$, and the theorem holds for this case.

Therefore we may assume that $\nu(G) = s$, i.e. G has a perfect matching. We prove the theorem by the induction on $\nu(G)$. If $\nu(G) = 1$, then G is is a graph with one edge, and then the assertion is trivial.

Assume that $\nu(G) > 1$. By Lemma 2.3, there is a vertex $x \in V(G)$ such that $\deg_G(x) = 1$. Set $xy \in E(G)$ (see Figure 3). Then, G_x has a perfect matching and $\nu(G_x) = \nu(G) - 1$.

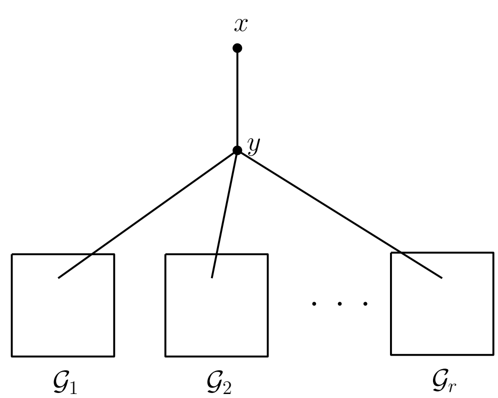

Fig. 3: *x is a leaf of G*

Let $G_1, \ldots, G_r$ be connected components of G_x. Then, they are sequentially Cohen-Macaulay by [14, Theorem 3.3]. Observe that each G_i has a perfect matching, bipartite and $s_i := \nu(G_i) \leq \nu(G_x) = \nu(G) - 1$. Then $\sum_{i=1}^{r} s_i = \nu(G_x) = \nu(G) - 1$. By the induction hypothesis, $\mathrm{m}(G_i) \leq 2^{\nu(G_i)-1} + 1$ for all i. We have

$$\mathrm{m}(G_x) = \prod_{i=1}^{r} \mathrm{m}(G_i) \leq \prod_{i=1}^{r} (2^{s_i-1} + 1). \tag{3.1}$$

Let $G_i' := G_i \setminus N_G(y)$. Since G is connected, $N_{G_i}(y) \neq \emptyset$. Hence, $\nu(G_i') \leq \nu(G_i) - 1 = s_i - 1$ because G_i has a perfect matching. Together with Lemma 2.6, it yields

$$\mathrm{m}(G_y) \leq \prod_{i=1}^{r} \mathrm{m}(G_i') \leq \prod_{i=1}^{r} 2^{\nu(G_i')} \leq \prod_{i=1}^{r} 2^{s_i-1}. \tag{3.2}$$

By Lemma 2.2(2) and inequations (3.1) and (3.2), we have

$$\mathrm{m}(G) = \mathrm{m}(G_x) + \mathrm{m}(G_y) \leq \prod_{i=1}^{r} (2^{s_i-1} + 1) + \prod_{i=1}^{r} 2^{s_i-1}.$$

By Lemma 3.1, $\mathrm{m}(G) \leq 2^{\sum_{i=1}^{r} s_i} + 1 = 2^{\nu(G)-1} + 1$.

Now, we prove the last assertion. If the equality holds, $r = 1$ or $\nu(G'_1) = \ldots = \nu(G'_r) = s_1 = \ldots = s_r = 1$. We next divide the rest of the proof into two cases:

Case 1: $\nu(G'_1) = \ldots = \nu(G'_r) = s_1 = \ldots = s_r = 1$.

Since $s_i = 1$, so G_i is one edge x_iy_i for all i. If $yx_i, yy_i \in E(G)$ for some i, then $G'_i = \emptyset$, a contradiction to $\nu(G'_i) = 1$. Since G is connected, we may assume that $yx_i \in E(G)$ and $yy_i \notin E(G)$ for all i. Therefore, $G \cong B(2, s-1)$ (see Figure 4).

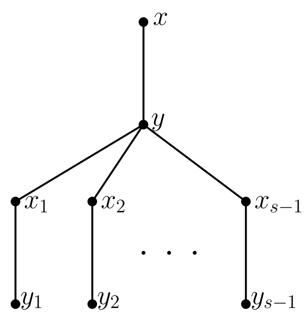

Fig. 4: *G Baton* $B(2, s-1)$

Case 2: $r = 1$.

Note that $G'_1 = G_y$. We have $\mathrm{m}(G_y) = \mathrm{m}(G'_1) = 2^{\nu(G'_1)}$ and $\nu(G_y) = \nu(G'_1) = s_1 - 1$. Thus, $\mathrm{m}(G_y) = 2^{\nu(G_y)}$. By Lemma 2.6, G_y is a Cameron-Walker bipartite graph. Let $\mathcal{H}_1, \ldots, \mathcal{H}_t$ be connected components of G_y. Then $\mathcal{H}_i$ is Cameron-Walker bipartite graphs for all i with partition $(K_i; T_i)$ such that there is at least one leaf edge attached to vertex in T_i. Let $\mathcal{H}_i$ is a star $(\{a_i\}; \{b_{i,1}, \ldots, b_{i,v_i}\})$ where $v_i \geq 1$ and $1 \leq i \leq s_1 - 1$. Let $K := \cup_{i=1}^{t} K_i$ and $T := \cup_{i=1}^{t} T_i$. Since G_y is a Cameron-Walker graph, $|T| = \nu(G_y) = s_1 - 1$.

Since G_1 has a perfect matching, $|V(G_1)| = 2s_1$. Then $n = 2s_1 + 2$. Let $N_G[y] = \{x, y, y_1, \ldots, y_u\}$. We have

$$\begin{aligned} |V(G_y)| \geq |K| + 2|T| &\Leftrightarrow n - |N_G[y]| \geq |K| + 2(s_1 - 1) \\ &\Leftrightarrow (2s_1 + 2) - (2 + u) \geq |K| + 2(s_1 - 1) \\ &\Leftrightarrow |K| + u \leq 2. \end{aligned}$$

Subcase 2.1. $|K| = 0$ and $u = 2$.

Indeed, we have

$$2s_1 = |V(G_1)| = 2 + (s_1 - 1) + \sum_{i=1}^{s_1-1} v_i \geq 2 + (s_1 - 1) + (s_1 - 1).$$

Hence, $v_1 = \ldots = v_{s_1-1} = 1$. Since G is a bipartite connected graph, we may assume that $y_1a_1 \in E(G)$ and $y_1b_{1,1} \notin E(G)$. Since $N_G(b_{1,1}) = \{a_1\}$ and $\{b_{1,1}, y_1\} \subseteq N_G(a_1)$, we have $\mathrm{m}(G) = \mathrm{m}(G_{a_1}) + \mathrm{m}(G_{b_{1,1}}) = 2^{s_1-1} + \mathrm{m}(G_{b_{1,1}}) \leq 2^{s_1-1} + 2^{s_1-1} = 2^{s_1}$, a contradiction.

Subcase 2.2. $|K| = 0$ and $u = 1$.

Indeed, if $v_1 = \ldots = v_{s_1-1} = 1$, then $\mathrm{m}(G) = \mathrm{m}(G_x) + \mathrm{m}(G_y) = 2^{s_1-1} + 2^{s_1-1} = 2^{s_1}$, a contraction. Therefore, without loss of generality, we may assume $v_1 \geq 2$. We have

$$2s_1 = |V(G_1)| = 1 + (s_1 - 1) + \sum_{i=1}^{t} v_i \geq 1 + (s_1 - 1) + s_1.$$

Hence, $v_1 = 2$ and $v_2 = \ldots = v_{s_1-1} = 1$. Since G is a bipartite connected graph, we may assume $y_1a_i \in E(G)$ and $y_1b_i \notin E(G)$ for all $2 \leq i \leq s_1 - 1$.

If $y_1a_1 \in E(G)$, then $y_1b_{1,1}, y_1b_{1,2} \notin E(G)$, a contradiction to the fact that G_x has a perfect matching. Thus, $y_1a_1 \notin E(G)$.

If $y_1b_{1,1}, y_1b_{1,2} \in E(G)$, then $\mathrm{m}(G) = \mathrm{m}(G_x) + \mathrm{m}(G_y) = \mathrm{m}(G_x) + 2^{s_1-1} = 2^{s_1}$, a contradiction. Therefore, we assume that $y_1b_{1,1} \in E(G)$ and $y_1b_{1,2} \notin E(G)$. Hence, $G \cong B(4, s-1)$.

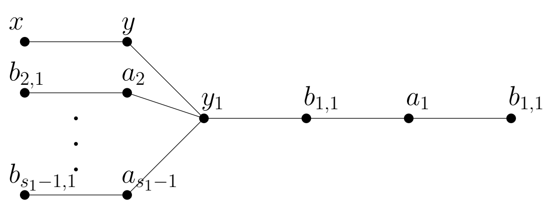

Fig. 5: *Baton* $B(4, s-1)$

Subcase 2.3. $|K| = 1$ and $u = 1$.

Assume $\alpha \in K$ and $\alpha a_i \in E(G)$ for all $i = 1, \ldots, j$ and $\alpha a_i \notin E(G)$ for all $i = j+1, \ldots, s_1 - 1$. We have

$$2s_1 = |V(G_1)| = 1 + 1 + (s_1 - 1) + \sum_{i=1}^{s_1-1} v_i \geq s_1 + 1 + (s_1 - 1) = 2s_1$$

Hence, $v_1 = \ldots = v_{s_1-1} = 1$. Since G is a bipartite connected graph, $y_1a_i \in E(G)$ and $y_1b_i \notin E(G)$ for all $i = 1, \ldots, j$.

If $y_1\alpha \in E(G)$, then $y_1a_i \notin E(G)$ for all $i = 1, \ldots, j$. This implies that $y_1b_i \notin E(G)$ for all $i = 1, \ldots, j$. Hence $G \cong B(2, s-1)$.

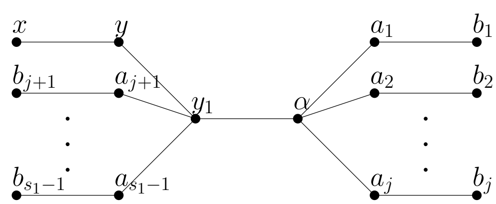

Fig. 6: *Baton* $B(2, s-1)$

If $y_1\alpha \notin E(G)$, then $y_1b_1 \in E(G)$. Thus, $y_1a_1 \notin E(G)$. Then $y_1a_i \notin E(G)$ for all $2 \leq i \leq j$. Therefore, $G \cong B(4, s-2)$.

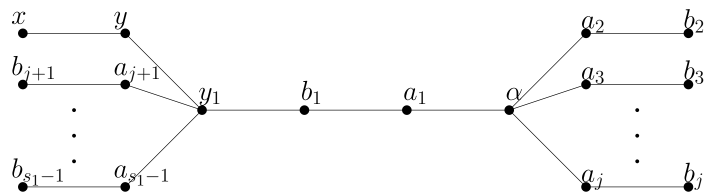

Fig.7: *Baton B(4,s-2)*

□

If the graph G is not sequentially Cohen-Macaulay, the theorem is not true as the following example.

Example 3.3. *Let* G *be a cycle graph of length* 8*. Then* G *is bipartite and* $|V(G)| = 8 = 2\cdot 4$*. In this case,* $\mathrm{m}(G) = 10 > 2^{4-1} + 1 = 9$.

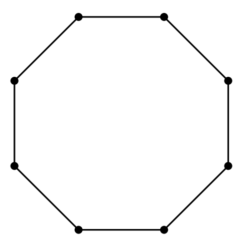

Fig. 8: *The cycle graph of length* 8*.*

If G be a tree, then

$$\mathrm{m}(G) \leq f(n) = \begin{cases} 2^s & \text{if } n = 2s+1, \\ 2^{s-1}+1 & \text{if } n = 2s. \end{cases}$$

Furthermore, $\mathrm{m}(G) = f(n)$ if and only if

$$G \cong \begin{cases} B(1,s) & \text{if } n = 2s+1, \\ B(2,s-1) \text{ or } B(4,s-2) & \text{if } n = 2s. \end{cases}$$

Proof. Since G is sequentially Cohen-Macaulay by [14, Theorem 2.13], so that the corollary follows from Theorem 3.2. □

REFERENCES

[1] J. A. Bondy, U. S. R. Murty, *Graph Theory, no. 244 in Graduate Texts in Mathematics*, Springer, 2008.

[2] K. Cameron, T. Walker, “The graphs with maximum induced matching and maximum matching the same size,” *Discrete Math.*, **299** 49-55, 2005.

[3] G. J. Chang and M. J. Jou., "The number of maximal independent sets in connected triangle-free graphs," *Discrete Math.*, 197/198:169-178, 1999.

[4] C. A. Francisco and H. T. Ha, "Whiskers and sequentially Cohen-Macaulay graphs," *J. Combin. Theory Ser. A.* **115**, 304-316, 2008.

[5] C. A. Francisco and A. Van Tuyl, "Sequentially Cohen-Macaulay edge ideals," *Proc. Amer. Math. Soc.*, **135**, 2327-2337, 2007.

[6] J. Herzog and T. Hibi, "Distributive lattices, bipartite graphs and Alexander duality," *J. Algebraic Combin.*, **22**, 289-302, 2005.

[7] T. Hibi, A. Higashitani, K. Kimura and A. B. O'Keefe, "Algebraic study on Cameron -Walker graphs," *J. Algebra*, **422**, 257-269, 2015.

[8] D. T. Hoang and T. N. Trung, "Coverings, matchings and the number of maximal independent sets of graphs," *Australas. J. Combin.,* 73, 424-431, 2019.

[9] M. Hujter, Z. Tuza, "The number of maximal independent sets in triangle-free graphs," *SIAM J. Discrete Math.,* **6** 284-288, 1993.

[10] J. Liu, "Maximal independent sets in bipartite graphs," *J. Graph. Theory,* **17**, 495-507, 1993.

[11] J. W. Moon and L. Moser, "On cliques in graphs," *Israel J. Math.,* **3**, pp. 23-28, 1965.

[12] B. E. Sagan, "A note on independent sets in trees," *SIAM J. Discrete Math.,* **1**, 105-108, 1988.

[13] R. P. Stanley, *Combinatorics and commutative algebra, Second edition,* Progress in Mathematics 41, Birkhuser, 1996.

[14] A. Van Tuyl and R. H. Villarreal, "Shellable graphs and sequentially Cohen-Macaulay bipartite graphs," *J. Combin. Theory, Ser. A,***115**, 799-814, 2008.

[15] H. S. Wilf, "The number of maximal independent sets in a tree," *SIAM J. Algebraic Discrete Methods*, **7**, 125-130, 1986.

TÓM TẮT

SỐ CÁC MẶT CỰC ĐẠI THUỘC CÁC ĐỒ THỊ COHEN-MACAULAY DÃY HAI PHẦN

Đào Thị Thanh Hà
Trường Đại học Vinh
Ngày nhận bài 05/5/2021, ngày nhận đăng 19/7/2021

Chúng tôi chặn số các tập độc lập cực đại thuộc các đồ thị Cohen-Macaulay dãy hai phần và đặc trưng hoàn toàn được các đồ thị giúp có chặn đúng.

Từ khóa: Đồ thị; tập độc lập; Cohen-Macaulay dãy.